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ABSTRACT

Studies have shown that students, after having studied physics concepts and being familiar with them for an appreciable time, may nevertheless lack the ancillary knowledge needed to use such concepts reliably; correspondingly, they exhibit major misconceptions and errors. Provided in this paper is an analysis of the ancillary knowledge required to make a scientific concept of principle effectively usable. Property concepts are addressed since these are centrally important to descriptions needed in science. This analysis includes, as a subset, the ancillary knowledge for a simple entity concept. Furthermore, the ancillary knowledge of a property concept is essentially the same as that for a principle. The most important ancillary knowledge required to make a concept effectively usable is that required to interpret the concept appropriately. This analysis of the ancillary knowledge needed for concept interpretation points out some practical implications for the learning and teaching of scientific concepts/principles. Students could be made aware of the ancillary knowledge (focusing on specification of concept, concept values, independent variables; instantiation; and error prevention) required to interpret a particular concept of interest or for use as a general skill in effectively learning any newly encountered concept or principle. (JN)

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Acquiring an Effective Understanding of Scientific Concepts

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In quantitative sciences, such as physics, special concepts and associated principles are logically the basic building blocks of the knowledge used to deduce important consequences, make predictions, and solve problems. However, mere definitions of concepts or statements of principles are psychologically far too primitive building blocks to permit the performance of complex intellectual tasks.

To be functionally useful, a conceptual building block (or "concept schema") must include a concept accompanied by the ancillary knowledge needed to make the concept effectively usable. In particular, this knowlege must be sufficient to ensure that the concept can be used reliably, i.e., without errors or ambiguities; easily and rapidly, so that use of the concept leaves adequate attention and time available to deal with other aspects of complex tasks; and flexibly, so that the concept can be used reliably in diverse and unfamiliar contexts. Similar comments can be made about a principle relating previously defined concepts.

The ancillary knowledge, required to make a concept or principle effectively usable, is far from trivial. Striking evidence supporting this statement comes from several recent studies.[1-7] These show that many students, after having studied physics concepts and been familiar with them for an appreciable time, may nevertheless lack the ancillary knowledge needed to use such concepts reliably. Correspondingly, they exhibit major misconceptions and errors.

The preceding comments indicate the importance of analyzing and explicating the ancillary knowledge required to make a scientific concept or principle affectively usable. Such an analysis, discussed in this paper, is interesting and useful from several points of view:

(1) From a scientific or psychological point of view, such an analysis helps make explicit underlying knowledge which is necessary (although not sufficient) for any scientific problem solving.[8,9] It also helps reveal important knowledge which is often "tacit", i.e., which is possessed by experts without their conscious awareness of its existence. Finally, such an analysis can help to predict many of the difficulties and errors exhibited by inexperienced students.

(2) From the practical perspective of teachers, such an analysis can help to identify important knowledge essential to students' understanding and learning of concepts or principles. Accordingly, it can be useful for diagnosing and minimizing the difficulties experienced by many students. Furthermore, it can provide the basis of explicit instructional methods for teaching concepts or principles more effectively.

(3) From the practical perspective of students, such an analysis can provide guidelines for studying concepts more effectively and can thus help students to acquire some important general learning skills.

As the analysis in the following pages indicates, the basic ancillary knowledge required to make a concept or principle effectively usable is remarkably large (although it is commonly possessed by any expert). This is one reason why the learning of a new scientific concept is a difficult task for students.

Kinds of Concepts and Associated Ancillary Knowledge

The simplest kind of concept is a particular "entity" (e.g., "the sun"). Any one member of a specific set of such particular entities (e.g., a "triangle", a "particle", ...) is then a "generic concept" or "variable".

A "property" is a more complex kind of concept used to describe one or more other concepts called the "independent variables" described by the property. This description is achieved by associating a unique value of the property for any possible set of values of the independent variables.[10] (If the property is a "quantity", its values are numbers; otherwise they may be members of any other specified set.) For example, "area" is a property describing a surface by associating a particular value (a positive real number) to any member of a set of entities called "surfaces". Similarly, "color" is a property describing an object by associating a particular value (one of the set of concepts "red", "yellow", "green", ...) to any object. As a last example, "velocity" is a concept describing jointly a particle, a reference frame, and a time by associating a vectorial numerical value to any particle for any reference frame and for any time. The particular independent variables described by a property are indicated by preceding prepositions: For example, one speaks of the area <u>of</u> a surface; or of the color <u>of</u> an object; or of the velocity <u>of</u> a particle relative <u>to</u> a particular reference frame <u>at</u> some specified time.

The discussion in the following pages will deal predominantly with property concepts since these are centrally important to provide the descriptions needed in any science. The analysis of the ancillary knowledge required to make a property concept effectively usable includes, as a subset, the ancillary knowledge for a simple entity concept. Furthermore, as discussed later, the ancillary knowledge for a property concept is essentially the same as that for a principle.

The most important ancillary knowledge, required to make a concept effectively usable, is that required to interpret the concept appropriately. This knowledge, summarized in Table 1 and discussed in the next three sections, includes that needed to specify the concept, to achieve this specification in various particular instances, and to do this without committing errors of interpretation. The other kinds of ancillary knowledge (e.g., knowledge about basic implications, knowledge about alternative symbolic representations, and guidelines about when and how to use the concept) will not be discussed further in this paper. Instead, the analysis of the ancillary knowledge needed for concept interpretation will be used to point out some practical implications for the teaching of scientific concepts or principles.

Insert Table 1 about here

Specification Knowledge

As indicated in Table 1, the most basic knowledge required to interpret a scientific concept is that needed to specify the concept fully and unambiguously. The important components of this "specification knowledge" are now discussed in turn.

Specification of a Concept

Ultimately the meaning of any scientific concept must be specified by explicit rules (e..g, definitions) which ensure that the concept is unambiguously identified so that it can lead to clearly interpretable scientific knowledge. The following ways of specifying a concept are <u>all</u> useful--summary descriptions because they are compact and easily remembered, informal descriptions because they clarify the essential meaning of a concept, and procedural specifications because they provide the most detailed specification.

<u>Summary description</u>. A summary description of a concept is useful because it provides a brief and precise statement of the meaning of the concept, a statement which can be easily remembered and used as the starting point for more complete elaborations. A typical example of such a summary description is the formal statement $\underset{\sim}{a} = d\underset{\sim}{v}/dt$ which defines compactly the concept "acceleration" (denoted by $\underset{\sim}{a}$) in terms of the velocity $\underset{\sim}{v}$ and the time t.

<u>Informal description</u>. An informal description of a concept is useful because it specifies the essential meaning of a concept without undue precision or excessive details. By focusing attention selectively on a few salient features, an informal description can help in relating a concept to more familiar knowledge and in retrieving the concept in complex situations. Indeed, such qualitative informal descriptions (and methods of successive refinement which proceed from qualitative to more detailed descriptions) can be very useful in facilitating problem-solving tasks.[8,9,11]

For example, the acceleration of a particle may be described informally by statements such as "acceleration is the rate of change of velocity with time" or "acceleration is a quantity describing the small change of a particle's velocity during a small time". Such statements are admittedly rather vague, but they make quite clear what essential quantities are interrelated by the property "acceleration" and when this property might be relevant.

Procedural specification. The preceding specifications of a concept, whether formal or informal, are descriptive (or "declarative"), i.e., expressed in terms of statements asserted to be true. A very important alternative way of specifying a concept is by means of a step-by-step procedure specifying how to identify or exhibit the concept. Such a "procedural specification" provides the most explicit and detailed specifications of a concept. It also has fundamental scientific importance as an operational definition which specifies what one must actually do to decide whether a concept is properly identified.

These remarks can be exemplified by the following procedural specification of the concept "acceleration": (1) Consider a specified particle P. (2) At some specified time t, consider the velocity of $\underset{\sim}{v}$ of P relative to some specified reference frame R. (3) For comparison, consider some neighboring time $t' = t + \Delta t$ and consider the velocity $\underset{\sim}{v}'$ of the particle P at this time. (4) Find the velocity change $\Delta\underset{\sim}{v} = \underset{\sim}{v}' - \underset{\sim}{v}$ by subtracting vectorially the old velocity $\underset{\sim}{v}$ from the new velocity $\underset{\sim}{v}'$. (See Fig. 1.) (5) Calculate the ratio $\Delta\underset{\sim}{v}/\Delta t$. (6) Verify that the time t' has been chosen sufficiently close to t so that a closer choice, making Δt smaller, would leave the ratio $\Delta\underset{\sim}{v}/\Delta t$ unchanged within the desired precision of description. In this case denote Δt by dt and $\Delta\underset{\sim}{v}$ by $d\underset{\sim}{v}$. (7) Identify the resulting ratio as the concept of interest and name it the "acceleration of P relative to R at the time t".

Insert Figure 1 about here

The preceding procedural specification makes abundantly clear the many complexities involved in the definition of the concept "acceleration", complexities which are largely hidden in the formal descriptive specification $\underset{\sim}{a} = d\underset{\sim}{v}/dt$. Indeed, the distinction between a procedural specification and a formal description is strikingly apparent in practice. For example, when students are asked to find the acceleration of a pendulum bob at the extreme position of its swing, where its velocity is zero, many students say that the acceleration is zero. Most of them continue to make this claim vociferously, even when they are specifically asked to use the definition of acceleration, written out explicitly as $\underset{\sim}{a} = d\underset{\sim}{v}/dt = (\underset{\sim}{v}'-\underset{\sim}{v})/(t'-t)$. But when these students are asked to follow the steps of the <u>procedure</u> specifying the acceleration, they change their minds and realize that the acceleration is non-zero. (Of course, experts are much more skilled in translating a formal description into a corresponding procedure.)

As another example, when novice students are asked to find the component of a vector $\underset{\sim}{V}$ along some specified direction $\underset{\sim}{i}$, most can easily answer this question when the direction $\underset{\sim}{i}$ is "horizontal", as shown in Fig. 2a. On the other hand, they often have difficulties in more general cases, such as that shown in Fig. 2b. But such difficulties disappear if students have learned the <u>procedure</u> specifying how to identify or find the component of a vector along some given direction. The reason is that such a procedure does <u>not</u> merely rely on the recognition of a familiar pattern. (Instead, it identifies the component by the general process of drawing, from the ends of the arrow representing the vector $\underset{\sim}{V}$, lines parallel and perpendicular to the given direction $\underset{\sim}{i}$.)

Insert Figure 2 about here

As the preceeding examples illustrate, it can be pedagogically very useful if students are asked to explain the meaning of a concept by specifying an appropriate procedure.

Applicability conditions. A detailed procedural specification helps make apparent the conditions under which a concept may legitimately be applied. Such applicability conditions must be made quite explicit to help avoid misinterpretations and errors.

For example, the concept "acceleration" can be applied to any particle, but not indiscriminately to any system of particles (a mistake sometimes committed by students). As another example, the concept "potential energy" must be accompanied by the applicability condition specifying that this concept can be used only for interactions described by conservative forces (i.e., forces which do work independent of the process between states of a system.)

Specification of Concept Values

The specification of a concept implies a corresponding specification of its values. Although such a knowledge about values is relatively simple, it needs to be made explicit if errors are to be avoided. Table 1 and the following paragraphs outline the most important knowledge about the values of a concept.

Value ingredients. The value of a concept is ordinarily specified by several ingredients, i.e., the elements needed to specify the type of value and the units needed for specification. For example, the concept "acceleration" has values which are vectors. The elements needed to specify this type of value are a "magnitude" and a "direction". The units are "meter/second^2".

In the case of value specification, as well as in more complex cases discussed later, the use of explicit symbolic expressions is an important aid to ensure correct usage of a concept. For then mere adherence to proper symbolic form (or "syntax") helps automatically ensure that a specification is complete and correct. For example, an appropriate symbolic expression for a value of the concept "acceleration" is "⟨magnitude with unit of length/time^2⟩ along ⟨direction⟩". Here anything enclosed between triangular brackets indicates a "slot" to be filled by an instance of the specified kind of entity. For example, a correct value specification of an acceleration might be "1.6 m/s^2 along the northern direction". By contrast, a value specification such as "1.6 m/s^2" would be incomplete and thus ambiguous because the slot about direction has not been filled in. Similarly, a value specification such as "1.6 m/s along the northern direction" would be incorrect because the slot for units has been filled by the wrong kind of unit.

Possible values. Proper value specification requires also knowledge about the domain of possible values of a concept (e.g., knowledge that the concept "kinetic energy" can assume all non-negative numerical values.) A knowledge of typical values is also valuable for making qualitative predictions and checking the solutions of problems. For example, it is useful to know the typical values of the acceleration have magnitudes of the order of a few meters/second^2 for falling objects or accelerating cars.

Specification of Independent Variables

Basic independent variables. The specification of a property concept implies a corresponding knowledge of all the basic independent variables needed to specify this property completely. Such knowledge can be subtle and needs to be made explicit to avoid likely errors and ambiguities.

For example, the concept "acceleration" is a property used to describe a particle at some particular time relative to some particular reference frame. Hence a complete specification of the concept "acceleration" requires a specification of all the following independent variables, namely "particle", "time", and "reference frame". Failure to specify any of these independent variables leads to ambiguities (i.e., no unique value could then be ascribed to the acceleration, nor could statements about this concept be judged true or false). For instance, the statement that "the acceleration of a ball at some instant is 10 m/s^2 downward" involves an incomplete specification of the acceleration because of failure to specify a reference frame. Thus the statement is ambiguous; e.g., it might be true if the earth is used as a reference frame, but false if the reference frame is an elevator moving relative to the earth.

An explicit knowledge of all the basic independent variables needed to specify a concept unambiguously is very important to the proper interpretation of a concept. (Indeed, deficiencies in such knowledge lead to many common confusions observed among students.) The use of explicit symbolic expressions is again a powerful aid for ensuring that a property concept is specified completely and correctly. For example, the word "acceleration", by itself, is really meaningless. Instead, the adequately defined concept is the one denoted by the full expression "the acceleration of ⟨particle⟩ at ⟨time⟩ relative to ⟨reference frame⟩", where each entity between angular brackets denotes a slot to be filled by a variable of the specified kind.

Consistent use of full symbolic or verbal expressions can greatly help students (and occasionally even experts) to avoid fuzzy thinking and thus to prevent many errors or confusions.[12] For example, talking about the "velocity of some ball at some particular time relative to some particular reference frame" focuses explicit attention on all relevant entities. On the other hand, when talking blithely about the "velocity of a ball", students are often lead to assume inappropriately that the velocity is relative to the earth (since specification of a reference frame has been ignored) or to assume inappropriately that the velocity is constant (since specification of a particular time has been ignored).

Another example, illustrating the importance of complete specifications, is provided by the concept of "force". In physics this concept is used to describe the interaction between particles and requires, therefore, the specification of at least <u>two</u> particles. Accordingly, the symbolic expression for force is of the form "force on ⟨particle⟩ by ⟨other particle⟩" where it is essential that <u>both</u> slots be properly filled. Indeed, to help students avoid errors and confusions, it is very useful to insist that students never use the word "force" unless followed by the phrase "<u>on</u> ... <u>by</u> ...". Insistence upon use of this full expression avoids the lay conception of force as an intrinsic property inherent in an object, as expressed by phrases such as "force <u>of</u> an object". It helps to avoid confusions between "action" and "reaction" if these historically hallowed words are discarded in favor of the much clearer expressions "force on A by B" and "force on B by A". It also helps to avoid students' inappropriate invocation of non-existing "centripetal" or "centrifugal" forces produced by no discernible objects.

Relevant properties of independent variables. As indicated in Table 1, it is important to know not only which basic independent variables are needed to specify a given concept, but also which particular properties of these variables are (or are not) required for a complete specification. For example, as mentioned previously, the basic independent variables needed to specify a "force" are the particle on which the force acts and the particle by which it is exerted. But not all properties of these particles are relevant to this specification. For instance, the positions of the particles are relevant and must be specified. On the other hand, the colors of these particles are irrelevant, as are their velocities (for ordinary central forces).

Note that the preceding knowledge, needed to explicate what particular parameters are (or are not) relevant to a specification of a given concept, is far from trivial. Indeed, it implies important understanding of functional dependencies or invariances in situations where the concept is pertinent.

Instantiation

In principle, the knowledge required to specify a concept adequately, as discussed in the preceding section, is sufficient to interpret the concept. But this knowledge, although essential, is too general and abstract to make the concept effectively usable in practice. Thus it is also necessary to know how to "instantiate" the concept, i.e., how to apply the concept reliably in various possible kinds of specific instances. (Indeed, it is a familiar fact that many students, even when able to state the definition of a concept, may be quite unable to apply this definition in particular cases.)

As indicated in Table 1, the knowledge needed to instantiate a concept involves the ability to do the following: (1) To identify or exhibit the concept for various possible values (or relative values) of the independent variables or of their properties. (2) To do this in various possible symbolic representations, e.g., in words, in pictures (diagrams or graphs), or formal mathematical symbolism.

For example, the acceleration $\underset{\sim}{a}$ (defined by $\underset{\sim}{a} = d\underset{\sim}{v}/dt$) involves a comparison of the velocity $\underset{\sim}{v}$ of a particle at some specified time t and of its velocity $\underset{\sim}{v}' = \underset{\sim}{v} + d\underset{\sim}{v}$ at a slightly later time $t' = t + dt$. Adequate instantiation knowledge then requires the ability to apply the concept "acceleration" in the following kinds of cases, described verbally as well as pictorially: (a) The new velocity $\underset{\sim}{v}'$ has the same direction as the original velocity $\underset{\sim}{v}$, but a larger or smaller magnitude, as indicated in Figures 3a and 3b. The acceleration $\underset{\sim}{a}$ has then, respectively, either the same or opposite direction compared to the velocity $\underset{\sim}{v}$. (b) The new velocity $\underset{\sim}{v}'$ has the same magnitude as $\underset{\sim}{v}$, but a different direction, as indicated in Figure 3c. The acceleration has then a direction perpendicular to the velocity $\underset{\sim}{v}$. (c) In the most general case, the new velocity $\underset{\sim}{v}'$ differs from $\underset{\sim}{v}$ in both magnitude and direction, as indicated in Figure 3d. The acceleration has then a direction not parallel to the velocity $\underset{\sim}{v}$, but toward the concave side of the particle's path.

Insert Figure 3 about here

Being able to identify and use various possible instances of a concept is sometimes far from trivial. For example, it often takes students a long time to understand that the innocent-looking definition $\underset{\sim}{a} = d\underset{\sim}{v}/dt$ of the concept "acceleration" encompasses all the various cases illustrated in Figure 3.

Error Prevention

Human beings are prone to errors. The reliable interpretation of a concept requires, therefore, also adequate knowledge to prevent errors i.e., knowledge to avoid likely errors, to detect such errors when they have been committed, and to correct them appropriately.

As indicated in Table 1, such error-prevention knowledge includes explicit warnings or "caveats" about errors likely to occur in the application of the concept; knowledge about how to discriminate any such error from the correct situation; and the use of explicit symbolism designed to help avoid such errors.

Warnings about Likely Errors

Reliable performance on any task is obviously facilitated if one is explicitly forwarned about likely errors and pitfalls. Such errors may be identified by actual observations of commonly made errors. A theoretically more interesting approach is to use an a-priori analysis to predict many of the kinds of errors likely to occur in the use of any newly encountered concept. Such an analysis must take into account the characteristics of the particular concept, i.e., the previously discussed knowledge required for the specification of the concept. It must also take into account the characteristics of the person using the concept, including the persons' preexisting knowledge. The results of such an analysis are briefly outlined in Table 2 which indicates some of the most common basic errors likely to occur in the application of any concept.[13]

Insert Table 2 about here

The likely errors listed in Table 2 correspond to errors in the various kinds of specification knowledge summarized in Table 1. The following paragraphs discuss and exemplify the most likely of these errors. The first two of these are gross confusions which result if a concept is identified by relying merely on the recognition of some salient features, rather than by applying explicitly the rules specifying the meaning of the concept.

Confusion of a concept with another concept denoted by a similar symbol (including lay terminology). Such a confusion occurs because a superficial similarity of symbols causes a failure to discriminate between different concepts. For example, the scientific concept "acceleration" (denoting the vector dv/dt describing the vectorial change of velocity) is likely to be confused with the lay term "acceleration" (used in everyday life to denote roughly the rate of increase of speed with time). As another example, concepts such as "kinetic energy", "potential energy", and "energy" may easily be confused because their names all include the same work "energy".

Confusion of a concept with another concept describing a different feature of the same situation. Such a confusion is caused by a failure to discriminate between related concepts which occur frequently in the same context. For example, "acceleration" and "velocity" are likely to be confused because both these concepts describe the motion of a particle, although different features of such motion.

Errors in specification rules. Even if a detailed rule or procedure is used to identify a concept, an error in some part of the rule can lead to misidentification of the concept. There may be many such possible errors since one or more steps in a specification rule may be omitted or wrong.

For example, the procedural specification of the concept "acceleration" involves a substraction $\underset{\sim}{v}' - \underset{\sim}{v}$ of velocities at slightly different times. If this *vectorial* subtraction is confused with a *numerical* subtraction of magnitudes, a wrong concept (the rate of change of speed dv/dt) is identified.

Errors in applicability conditions. An example of such an error would be the attempted use of a potential energy to describe interaction due to friction forces (since the concept of potential energy is only applicable in the case of conservative forces).

A particularly common error in applicability conditions occurs when a concept, describing a special case, is inappropriately extended to a more general case where it is not valid. Such confusions of special cases with general cases are particularly likely when the special case has an appealing simplicity and has been encountered first in one's learning experience. For example, students often encounter the concept "velocity" first in the simple special case of uniform motion along a straight line when the velocity may be simply defined by the numerical ratio s/t (where s is the distance traveled during the time t). It is then predictably likely that students will subsequently confuse this definition of the concept with the general concept of "velocity" defined as the *vector* d$\underset{\sim}{r}$/dt (where d$\underset{\sim}{r}$ is the *infinitesimal* displacement d$\underset{\sim}{r}$ during an *infinitesimal* time dt).

Errors in specification of values. Errors in the specification of the values of a concept occur when some of the ingredients necessary to specify a value are omitted or wrong, or because impossible values are attributed to the concept. Such errors are easy to avoid, although common among novice students. The following are examples os such errors: Describing the value of an acceleration by specifying a magnitude without a direction; specifying the value of a potential energy with the wrong unit "newton"; or stating that the value of a kinetic energy is negative.

Errors in specification of independent variables. A very common kind of error results from the omission of some of the independent variables required to specify a property concept. The consequences are an incomplete specification of the concept and concomitant ambiguities; these can often lead to troublesome confusions and seemingly perplexing paradoxes. The following are examples of such omissions: Talking about an acceleration without specifying the reference frame relative to which it is measured; talking about a potential energy without specifying the standard position from which it is measured; or talking about a force without specifying the object exerting this force.

Discriminations.

Table 2 and the preceding comments help to identify likely errors which must be avoided if a concept is to be used reliably. Hence it is essential to be able to discriminate between any such error and the correct application of the concept. To acquire the ability to make such discriminations while learning an unfamiliar concept, it is useful to compare explicitly the error (and its consequences) with the correct situation. Distinguishing features, characterized abstractly as well as exemplified in specific cases, can then be made explicitly apparent so that they can be readily recognized and heeded.

As an example, consider the error involving the confusion of the concept "acceleration" with the concept "velocity". Explicit comparison of these concepts leads to the knowledge needed to discriminate between them. In particular, the two concepts are characterized by the following distinguishing features: The acceleration describes the rate of change of velocity, whereas the velocity describes a rate of change of position; also the unit of acceleration is meter/second^2, whereas the unit of velocity is meter/second. Specific examples illustrating distinctions between these concepts are the following: The acceleration can be zero while the velocity is non-zero (e.g., for motion with constant velocity); the acceleration can be non-zero while the velocity is zero (e.g., at the highest point of a ball thrown vertically upward); and the acceleration can be constant while the velocity is changing (e.g., for a freely falling object).

A knowledge of such discriminations for each likely error is an important part of the ancillary knowledge needed to make a concept reliably usable.

Helpful Symbolism

A powerful aid for preventing errors is the introduction and use of appropriate symbolism; for then strict adherence to symbolic form can automatically help to avoid many errors.

As a trivial example, confusion between the concept "velocity" (a vector) and the concept "speed" (the magnitude of the velocity) can be minimized by consistently using the letter $\underset{\sim}{v}$ (printed in boldface type or underscored by a squiggly line) to denote the vector representing the velocity, while using the unadorned letter v to denote the number representing the speed.

Much more important examples of helpful symbolism involve the use of standarized symbolic expressions with "slots" indicating explicitly all the kinds of information that need be supplied. As previously discussed and exemplified, such symbolic expressions can be used to indicate explicitly all the ingredients needed to specify the value of a concept or all the independent variables needed to specify a property. Consistent use of such symbolic forms can greatly help to avoid many errors of omission or commission in the application of concepts.

Application to Principles

The preceding sections discussed at some length the ancillary knowledge needed to interpret concepts (e.g., properties such as "acceleration", "potential energy", ...). The preceding discussion can be readily extended to principles expressing important relations between previously defined concepts (e.g., the principle $\Delta K = W$ relating kinetic energy and work, or the gravitational force law $F = Gm_1 m_2/R^2$).

Indeed, any valid relation between concepts can be regarded as a "truth property" (or "predicate") which asserts that the property has the value "true" whenever the values of the concepts are related in some specified way. With minimal modifications, the ancillary knowledge needed to interpret a principle is thus the same as that outlined in Table 1 for any property concept.

Thus Table 1, when applied to a principle, asserts that the specification of the principle can be achieved by a formal summary description (such as an equation), by informal qualitative statements, or by a detailed procedure which specifies what must be done to determine that the specified principle is true. The specification of the value of a principle is trivial, i.e., this value is simply "true". The specification of independent variables includes again the specification of basic independent variables which need be specified and the specification of the relevant properties thereof. (For example, in the case of Newton's motion principle $m\underset{\sim}{a} = \underset{\sim}{F}$, the basic independent variables are some specified *particle*, some *other particle* with which it interacts, some specified *time*, and some specified *inertial reference frame*. The relevant properties of these independent variables are the mass m of this particle, its acceleration $\underset{\sim}{a}$ at this time relative to the specified reference frame, and the force $\underset{\sim}{F}$ on this particle by all other particles interacting with it.) These remarks should suffice to indicate that our entire previous discussion is equally applicable to concepts as well as to principles relating previously defined concepts.

Implications for Learning or Teaching

The preceding sections have sought to identify and explicate the ancillary knowledge required to interpret scientific concepts or principles. The discussion has made apparent that this ancillary knowledge is quite large and extends considerably beyond mere definitions of concepts or statements of principles. Such knowledge is commonly possessed by any expert, although he or she may not be consciously aware of its existence or able to articulate it explicitly. On the other hand, the acquisition of such knowledge by students is a demanding task.

The following paragraphs outline briefly the difficulties faced by students trying to learn unfamiliar concepts or principles. Then they explore the prospects of instructional methods exploiting the analysis of the preceding sections to teach concepts and prinicples more effectively.

Learning Difficulties

Anyone trying to learn an unfamiliar scientific concept or principle faces appreciable difficulties. Some of these are due to intrinsic characteristics of such scientific concepts or principles: (a) As discussed in the preceding sections, the knowledge required to interpret and apply such a concept or principle is considerable and sometimes subtle. (b) This knowledge often demands meticulous attention to details and requires fine discriminations to achieve the unabmiguity required for accurate scientific predictions.

Other difficulties are characteristic of the person in the role of student trying to learn new concepts or principles: (a) A student brings to a learning situation many concepts and principles acquired in daily life or from more formal prior learning experiences. Hence the student's pre-existing knowledge must be appropriately modified or transcended before new concepts or principles can be used without confusion and integrated into a new knowledge structure. (b) A student, unless thoroughly versed in scientific thinking, approaches learning from the vantage point of daily life where concepts or principles are adequately useful even if they are specified vaguely and somewhat inconsistently. Hence everyday concepts (e.g., "chair", "color",...) are often adequately specified by reference to prototypical cases which can be readily recognized or used for approximate comparisons. By contrast, scientific concepts need to be specified by explicit rules to ensure that they have unambiguous meanings. The learning of scientific concepts is thus a demanding task, rather different from the learning of concepts in daily life, and is correspondingly quite difficult for novice students unfamiliar with this mode of learning.

How effective are common teaching methods in dealing with these learning difficulties?

Methods commonly used to teach concepts or principles involve presenting a new concept or principle, exemplifying the concept or principle in some special cases, and then providing students with practice in applying the concept or principle in various situations. Through a process of trial-and-error learning, students then gradually learn to avoid mistakes and to use the concept or principle more reliably.

There is considerable evidence that such teaching methods are neither very efficient nor effective. Indeed, after formal instruction and after months (or even years) of using a scientific concept or principle, many students still exhibit gross misconceptions, confusions, and other persistent errors.[1-7] Furthermore, although students may nominally be familiar with certain concepts or principles, they often do not feel comfortable to use them spontaneously as intellectual tools facilitating their own thinking.

Teaching Applications

The analysis in the preceding sections indentifies various kinds of important ancillary knowledge required to make a concept or principle effectively usable. This analysis can be used as the basis for instructional methods which teach such ancillary knowledge explicitly. It can also help to diagnose the causes of students' observed errors and difficulties.

The following paragraphs outline some suggested teaching methods based on this analysis. Although these suggestions are tentative and based on limited evidence, they provide a systematic approach suitable for further study and improvement.

Teaching particular concepts. A very common instructional aim is to teach students particular scientific concepts or principles (e.g., particular concepts such as "acceleration"). The ancillary knowledge summarized in Table 1 can then be used by an instructor, textbook, or other instructional medium to make explicit the ancillary knowledge required to interpret the particular concept of interest. (For example, the instructor can identify what particular independent variables are necessary to specify fully the concept "acceleration"; or the instructor can identify the likely error caused by confusion between the concept "acceleration" and the concept "velocity".) Systematic instruction then involves teaching students explicitly these specific kinds of ancillary knowledge at the time when the unfamiliar concept is first encountered. Indeed, the entries listed in Table 1 can easily be converted into specific questions which any student should be able to answer about the particular concept (e.g., questions such as "what is the procedure used to specify the meaning of the concept acceleration?").

Not only must one ensure that students display explicit familiarity with the various kinds of ancillary knowledge about a concept, but also that they actually use this knowledge when applying a concept. (For example, students should spontaneously answer questions about the acceleration by applying the procedure used to define this concept.) It is advisable that students acquire and consolidate this ancillary knowledge about a concept in the context of relatively simple questions and exercises. Only afterwards should they be asked to apply the concept in more complex problems.

Effective use of a concept requires that the ancillary knowledge about the concept become ultimately intuitive and habitually used. Needless to say, this requires adequate practice, but the right kind of practice specifically suggested by the analysis of the concept. Furthermore, explicit awareness of this ancillary knowledge can be useful to students, even after a concept has become intuitively familiar, since such explicit knowledge helps to debug errors or to cope with novel situations.

I have recently tried to exploit some of these teaching guidelines in actual classroom situations. This experience indicates that explicit teaching approaches based on the analysis in this paper can be very useful in practice. For example, it is very helpful to ask students to verbalize and apply procedures for identifying concepts. It also helps avoid many confusions to insist that students use full verbal expressions (such as "force on what by what"). However, the implementation of teaching procedures based on such an explicit analysis reveals also particularly clearly some general issues and difficulties inherent in any teaching process, issues which are worthy of further study in their own right.

Teaching conceptual learning skills. The preceding comments have dealt with the teaching of particular concepts or principles. A much more ambitious instructional goal would involve teaching students the general skill enabling them to learn effectively any newly encountered concept or principle. The analysis presented in the preceding pages, as summarized in Table 1, is again basic to the systematic teaching of such a general learning skill. But now students would have to be taught the general ancillary knowledge required to make any concept or principle effectively usable, and would themselves have to translate this general knowledge into specific knowledge about any particular concept. This is clearly a much more difficult teaching task, but one of great importance. Indeed, successful implementation of such instruction would make students better independent learners who know explicitly what they need to study to achieve competent use of any new concept.

There is evidence that such instruction can be successfully implemented in practice. For example, a few years ago some collaborators and myself[14], using a rather rudimentary analysis of concept learning and some very primitive teaching methods based on this analysis, were able to show that students could be taught to become significantly better independent learners of new concepts. The more extensive analysis presented in the preceding pages, together with more explicit teaching methods, promises to lead to much more effective teaching of such general conceptual learning skills.

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Footnotes

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(10) A property is thus, in a generalized sense, a mathematical function associating a unique value of a variable to any set of values of some independent variables.

(11) Larkin, J. H. & Reif, F. Understanding and teaching problem solving in physics. European Journal of Science Education, 1979, 1, 191-203.

(12) A full symbolic expression may, of course, be abbreviated if this is done explicitly _after_ the omitted independent variables have been specified once and for all at the beginning of a discussion. For example, one may speak simply of the "velocity of the ball" _after_ one has specified that the reference frame is the laboratory.

(13) Different basic errors listed in Table 2 may sometimes lead to the same overtly observable error, i.e., the same observable error may sometimes be traceable to different underlying errors.

(14) Reif, F., Brackett, G. C., & Larkin, J. H. Teaching general learning and problem solving skills. American Journal of Physics, 1976, 44, 212-217.

Table 1

Interpretation of a concept

- *Specification*
  - Specification of concept
    - Summary description
    - Informal description
    - Procedural specification
    - Applicability conditions
  - Specification of concept values
    - Ingredients and symbolic expression (elements specifying type, units)
    - Possible values (and typical values)
  - Specification of independent variables
    - Basic independent variables and symbolic expression
    - Relevant properties of independent variables
- *Instantiation*
  - Various values of independent variables and of their properties
  - Various symbolic representations
- *Error prevention*
  - Warnings about likely errors (see Table 2)
  - Discrimination between each error and correct case
  - Helpful symbolism

Table 2

Likely Errors

- * Errors in specification of concept
    - * Gross confusions
        - * Confusion with concept denoted by similar symbol
        - * Confusion with concept describing different features of same situation
    - * Errors in specification rules
    - * Errors in applicability conditions
- * Errors in specification of values
    - * Errors in specifying ingredients
    - * Errors in possible values
- * Errors in specification of independent variables
    - * Omitted independent variables
    - * Wrong independent variables or properties thereof

Figure Captions

Figure 1. Diagram illustrating specification of the concept "acceleration".

Figure 2. Finding the component of a vector $\underset{\sim}{V}$ along a direction $\underset{\sim}{i}$.

Figure 3. Various instances of the concept "acceleration".